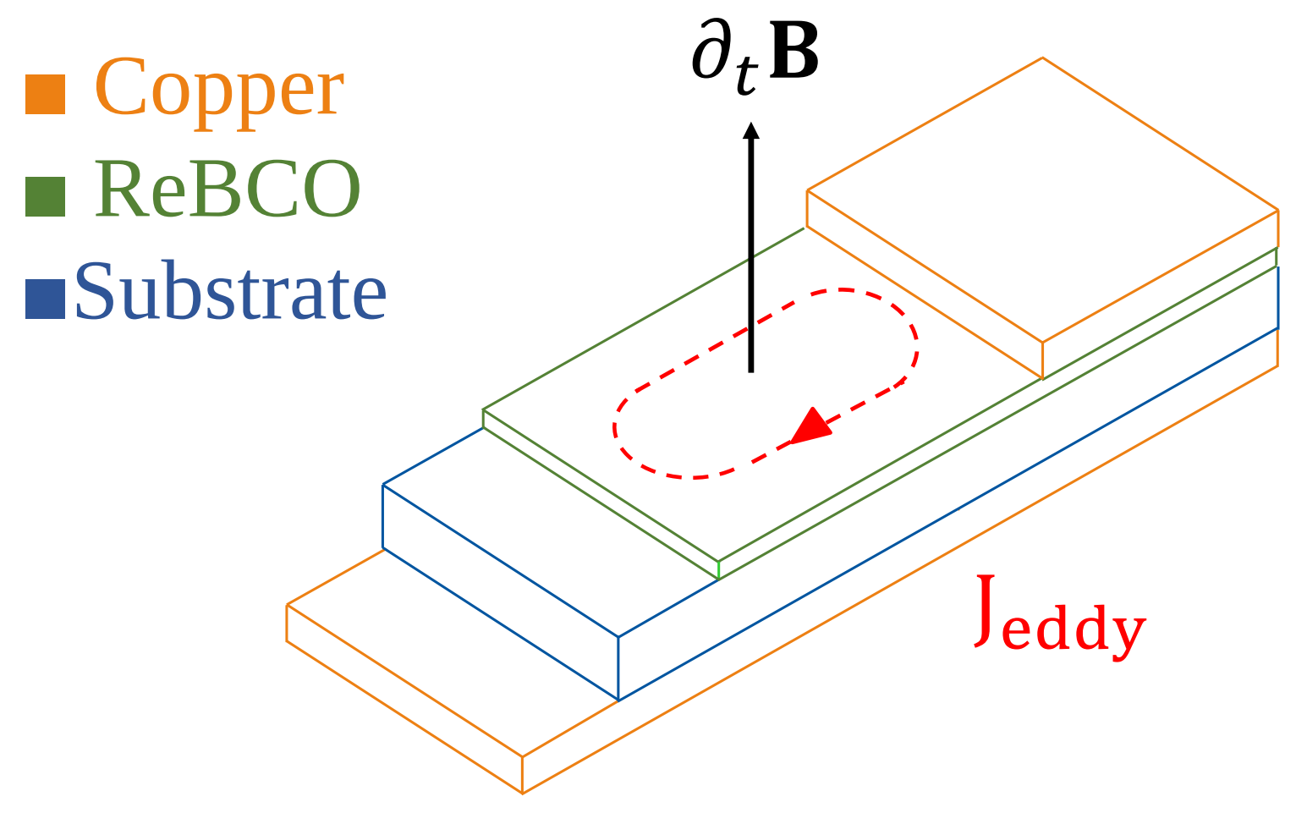
Coupled A-H Field Formulation for High-Temperature Superconducting Magnets in COMSOL Multiphysics®

L. Bortot[1,2], I. Cortes Garcia[1], H. De Gersem[1],
M. Mentink[2], J. Van Nugteren[2], S. Schöps[1], A. Verweij[2]
[1] Technische Universität Darmstadt, Germany
[2] CERN, Geneva, Switzerland

TECHNISCHE UNIVERSITÄT DARMSTADT
CERN
Contact: lorenzo.bortot@cern.ch

INTRODUCTION

Superconducting ReBCO Tape



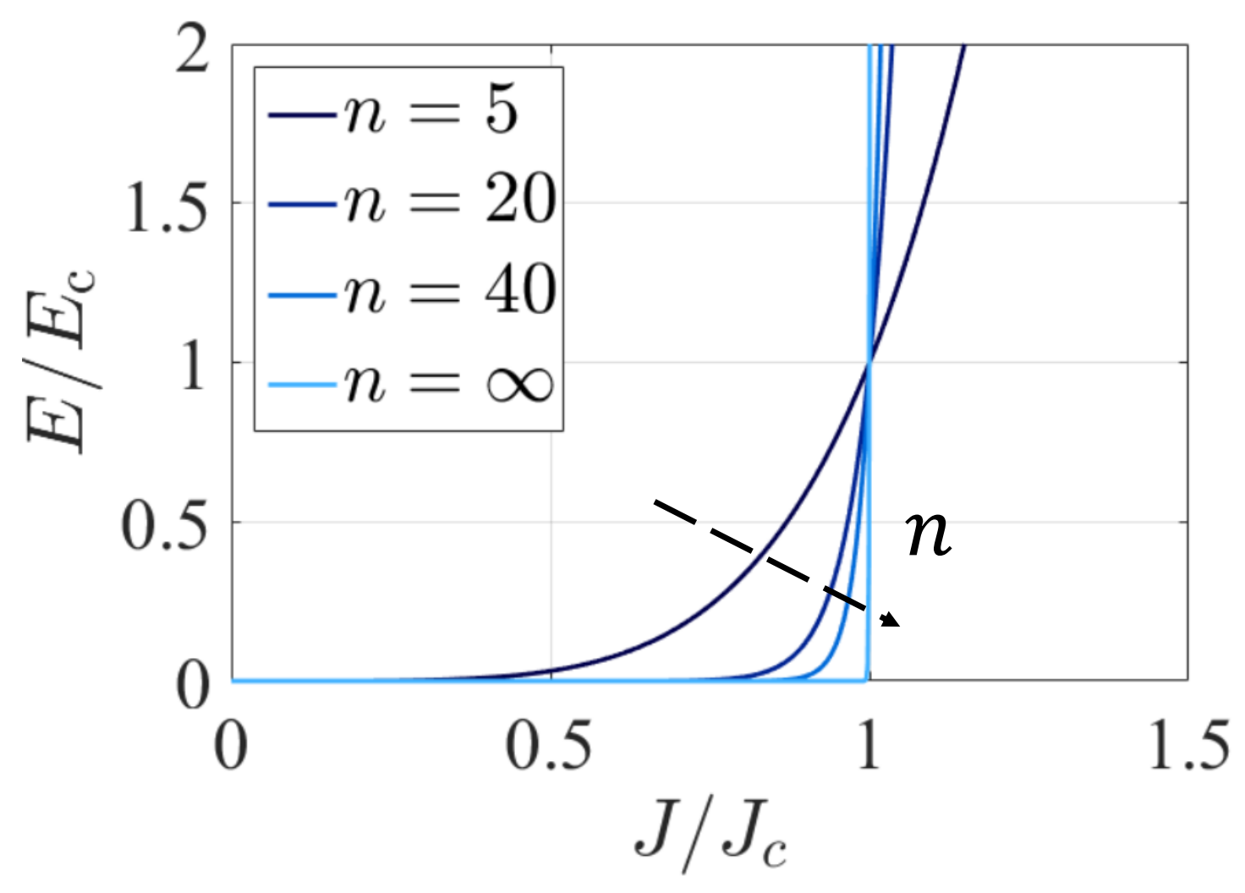
Screening currents J_{eddy} relevant for:

- Magnetic analysis (field quality)
- Thermal analysis (quench phenomena)

Multiphysics

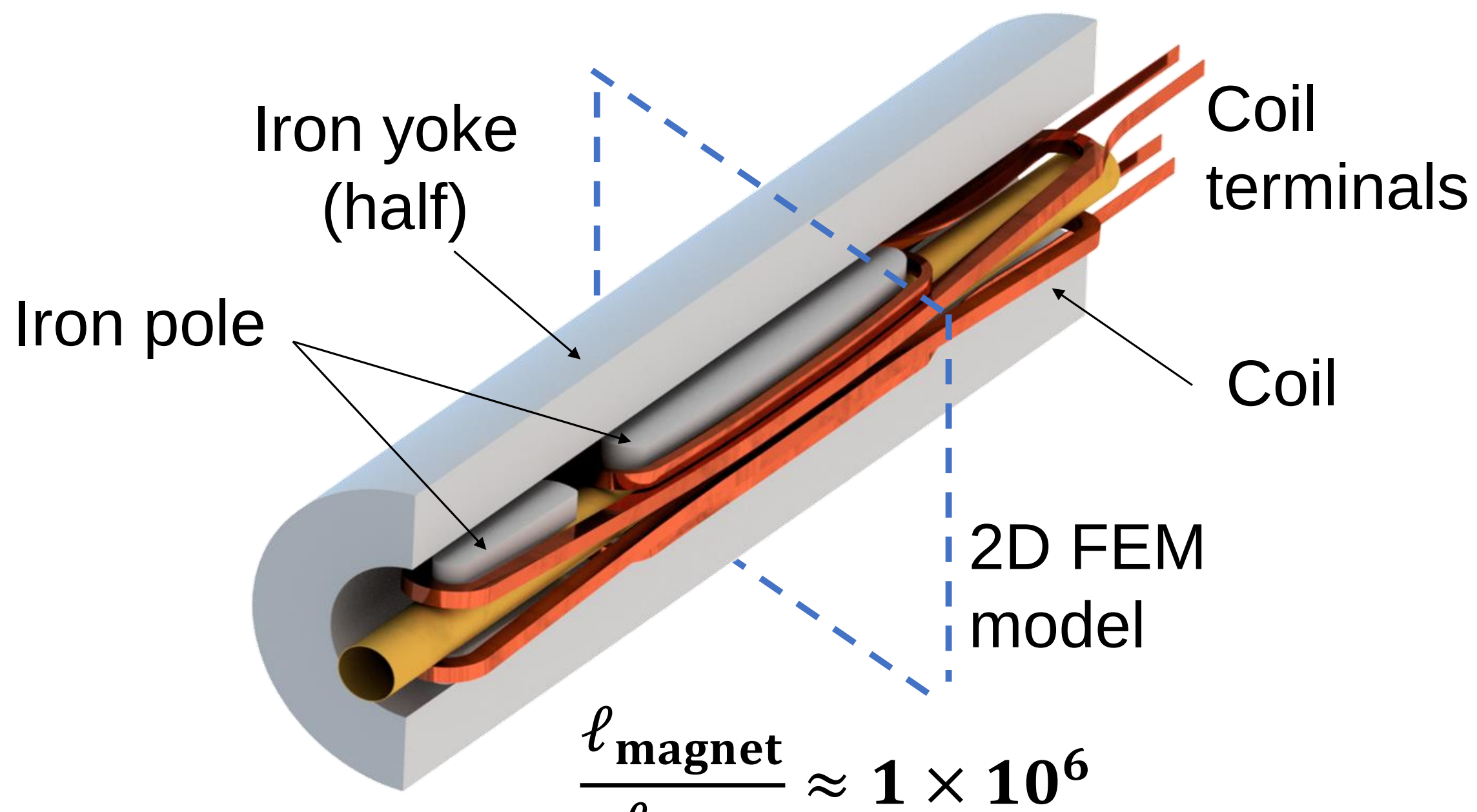
Constitutive Law for (E, J)

$$E = E_c\left(\frac{|\mathbf{J}|}{J_c(\mathbf{B},T)}\right)^n \quad \Bigg| \quad \begin{array}{l} E_c = 1e^{-4}\ V/m \\ 10 \le n \le 30 \end{array}$$



Highly nonlinear

Application:
Dipole insert magnet Feather M2 [1]

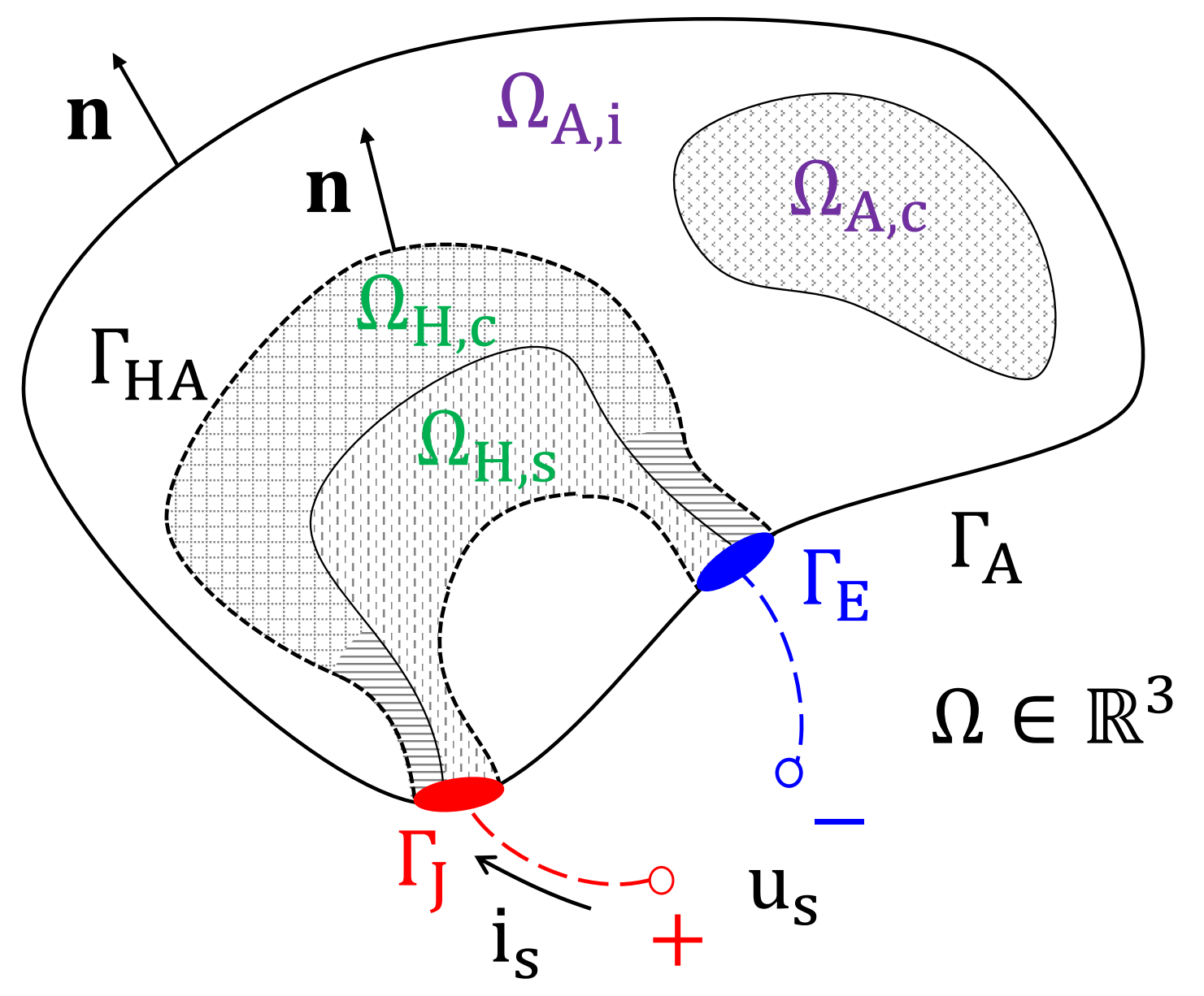


$$\frac{\ell_{magnet}}{\ell_{tape}} \approx \mathbf{1 \times 10^6}$$

Multiscale

FORMULATION

Domain Decomposition



$\Omega_H = \Omega_{H,s} \cup \Omega_{H,c}$ active region:

- $\Omega_{H,s}$ superconductors ($\rho \to 0$)
- $\Omega_{H,c}$ normal conductors

$\Omega_A = \Omega_{A,c} \cup \Omega_{A,i}$ passive region:

- $\Omega_{A,c}$ normal conductors
- $\Omega_{A,i}$ insulators ($\sigma \to 0$)

Vanishing resistivity in Ω_H ($\sigma \to +\infty$), conductivity in Ω_A ($\rho \to +\infty$)!
Finite material properties ensured by a coupled field formulation

Strong Formulation [2]

$$\nabla \times \rho \nabla \times \mathbf{H} + \mu \partial_t \mathbf{H} + \nabla \times \boldsymbol{\chi}\, u_s = 0 \qquad \text{in } \Omega_H$$
$$\nabla \times \nu\, \nabla \times \mathbf{A}^\star + \sigma \partial_t \mathbf{A}^\star = 0 \qquad \text{In } \Omega_A$$
$$\rho_m C_p \partial_t T - \nabla \cdot k \nabla T - \mathbf{J} \cdot \rho \mathbf{J} = 0 \qquad \text{in } \Omega$$

$$\int_{\Omega_H} \boldsymbol{\chi} \cdot \nabla \times \mathbf{H}\, d\Omega = i_s \qquad \text{Current constraint}$$
$$\boldsymbol{\chi} = -\nabla \xi, \qquad \xi : \ \nabla \cdot \sigma \nabla \xi = 0 \qquad \text{Voltage distribution function}$$

Boundary Conditions

$$\mathbf{A}^\star \times \mathbf{n} = 0 \quad \text{on } \Gamma_A$$
$$\mathbf{E} \times \mathbf{n} = 0 \quad \text{on } \Gamma_E, \Gamma_J$$
$$\nabla T \cdot \mathbf{n} = 0 \quad \text{on } \Gamma_A$$

Interface Conditions on Γ_{AH}

$$(\nu\, \nabla \times \mathbf{A}^\star - \mathbf{H}) \times \mathbf{n} = 0$$
$$(\partial_t \mathbf{A}^\star + \rho \nabla \times \mathbf{H} + \boldsymbol{\chi}\, u_s) \times \mathbf{n} = 0$$

Thin-Shell Approximation

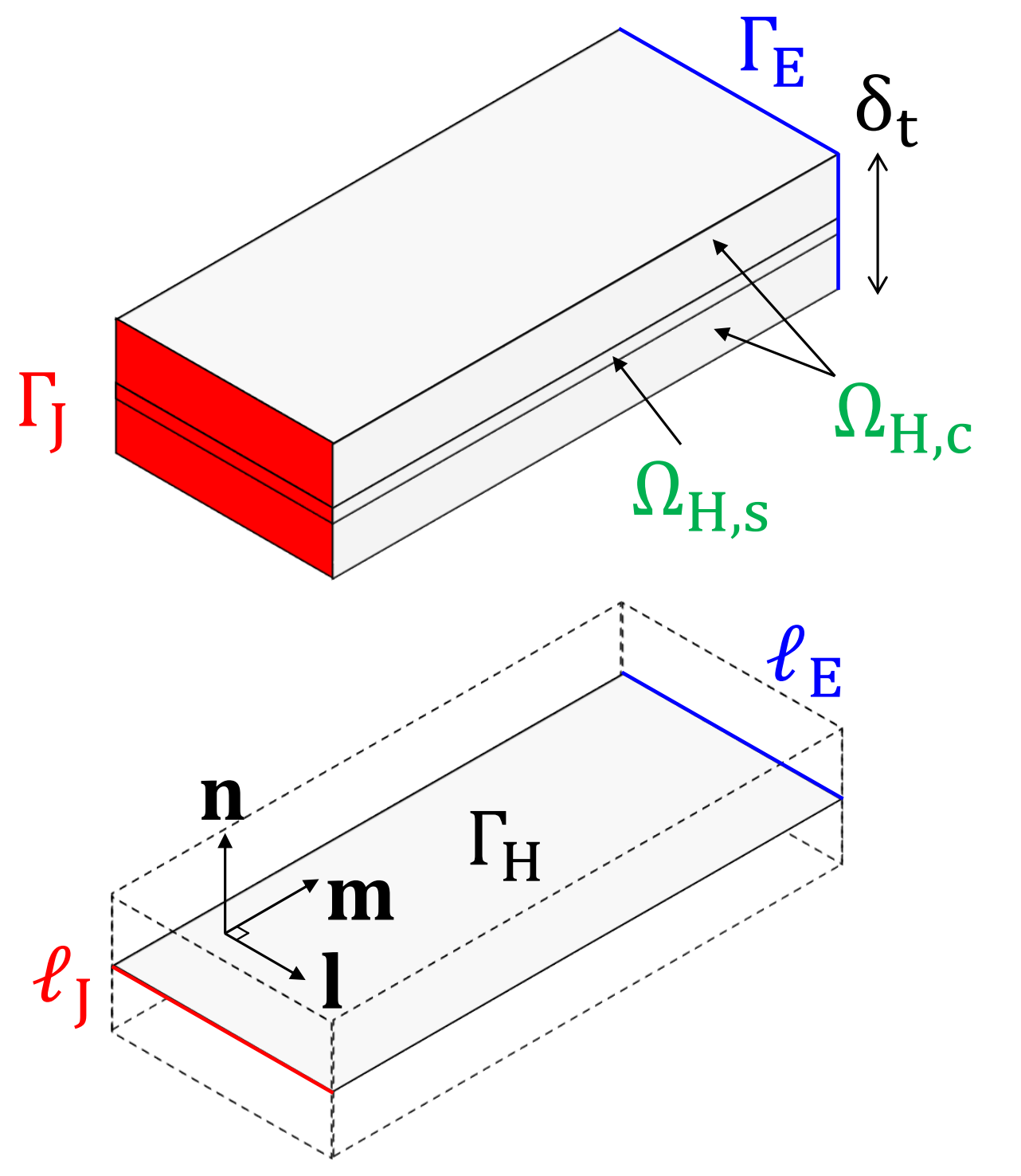
$\Omega_H \to \Gamma_H$ volume as slab
$\mathbf{J} \cdot \mathbf{n} = 0$ current in the slab
$\nabla_t \cdot = \left(\frac{\partial \cdot}{\partial_r}, \frac{\partial \cdot}{\partial_z}, 0\right)$ no variation along $\mathbf{n}$

Within the shell:

$$\nabla_t \times \rho \nabla_t \times \mathbf{H_n} + \mu \partial_t \mathbf{H_n} + \nabla_t \times \boldsymbol{\chi}\, u_s = 0$$
$$\mathbf{K} = \delta_t \nabla_t \times \mathbf{H_n}$$

At the shell interface:

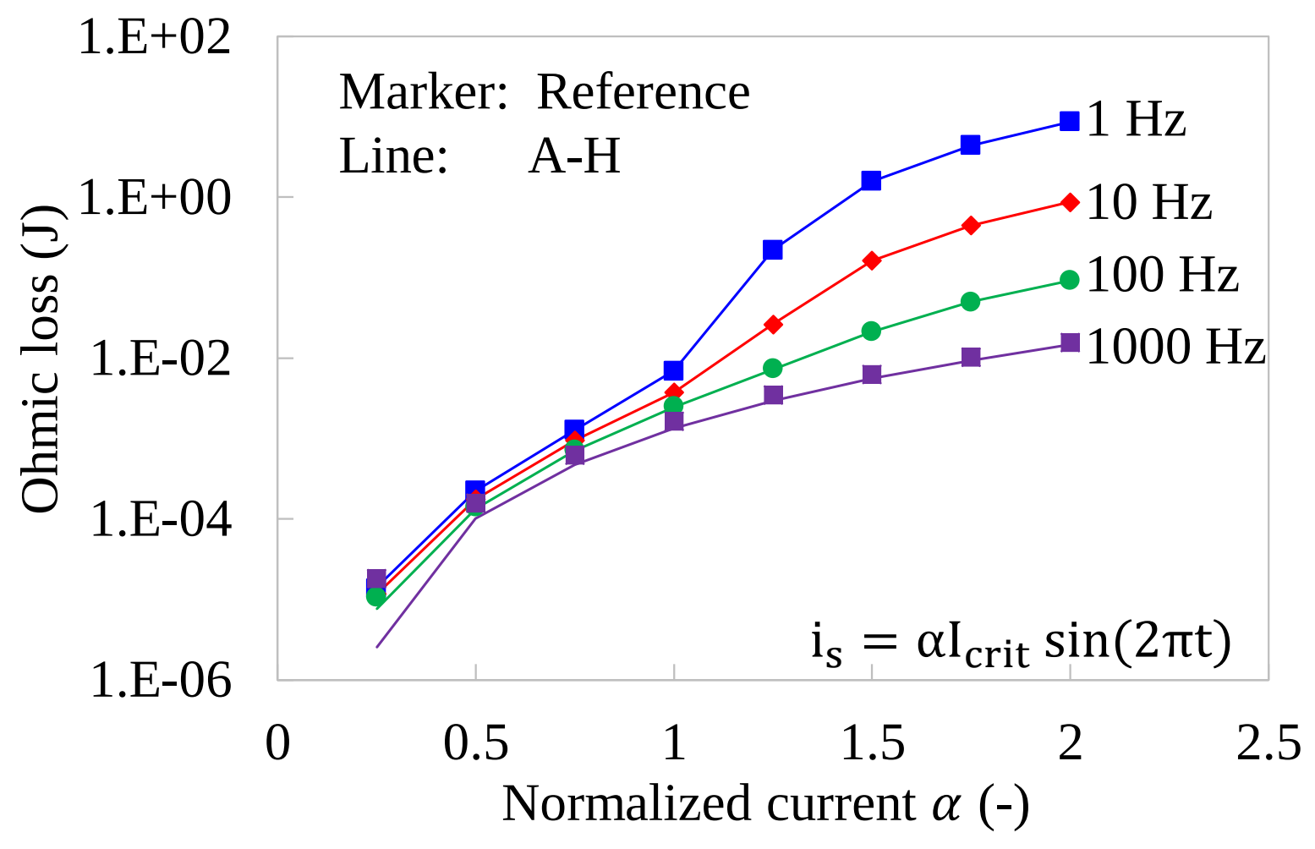
$$\mathbf{K} = [\nu(\nabla \times \mathbf{A}_1^\star - \nabla \times \mathbf{A}_2^\star)] \times \mathbf{n}$$



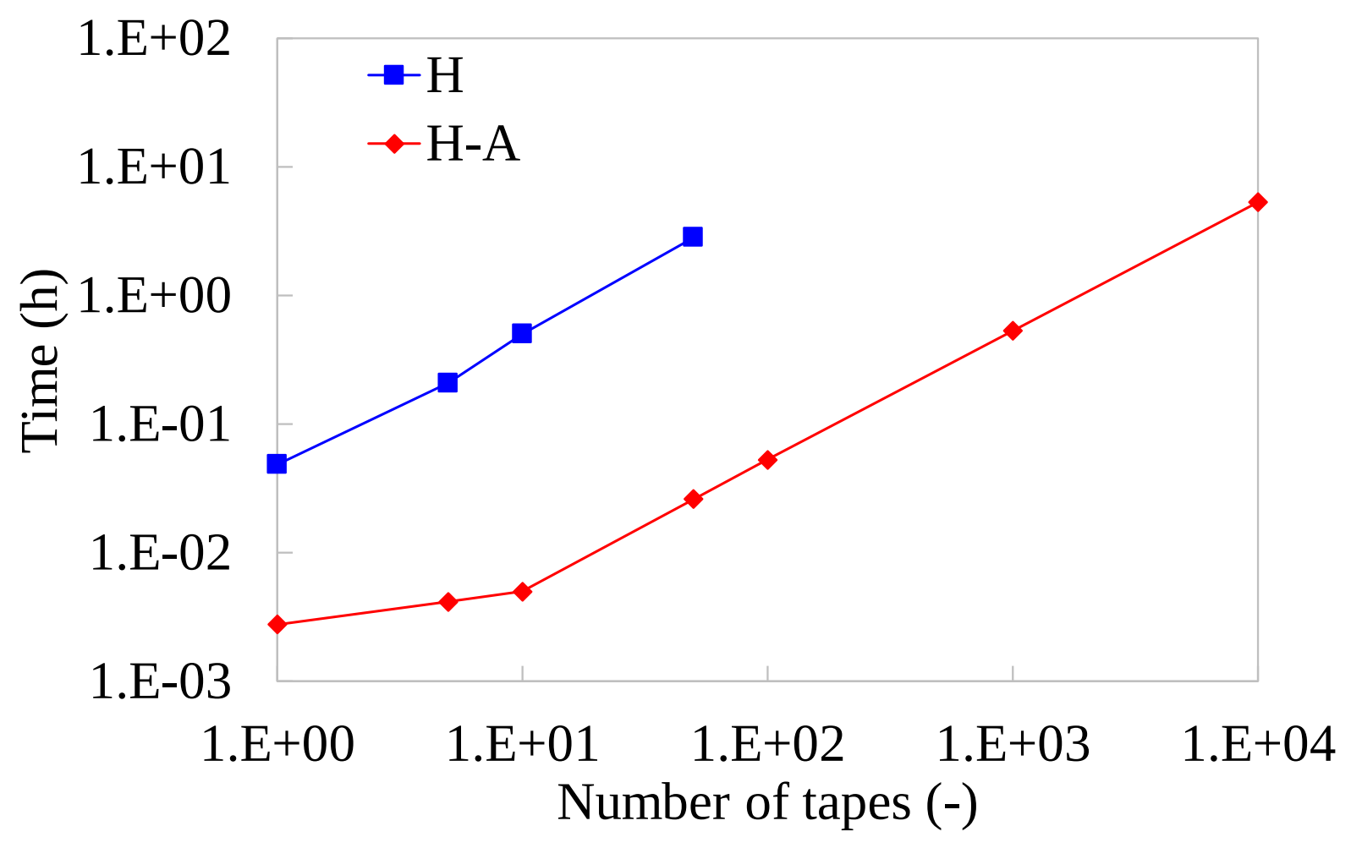
NUMERICAL RESULTS

Verification of the FEM Implementation in COMSOL

Crosscheck:
Single tape



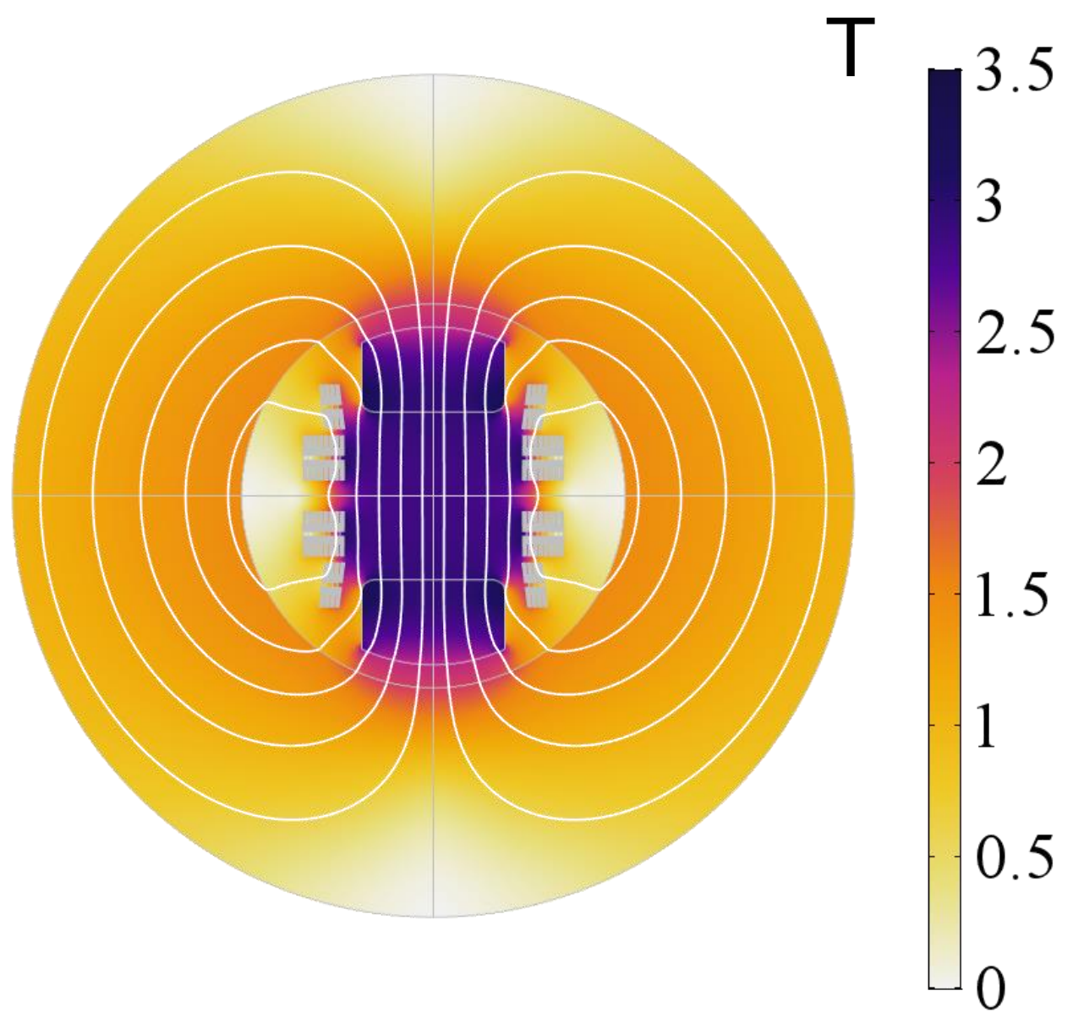
Benchmark:
H vs A-H formulations



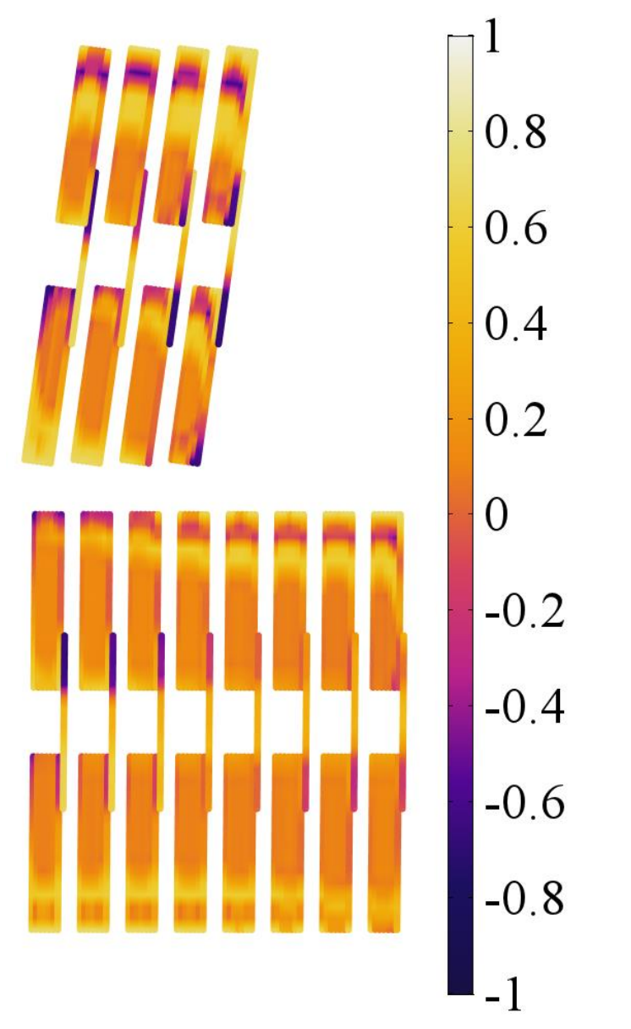
Reference model available at: HTS MODELLING WORKGROUP

Simulation of the Feather-M2 Magnet [3]


Feather-M2 coils (Courtesy of J. Van Nugteren)



Magnet flux density at 5 kA



Normalized current density in the coil, at 2 kA

Acknowledgements

This work has been funded by the Wolfgang Gentner Programme of the German Federal Ministry of Education and Research (grant no. 05E15CHA), and by the Graduate School CE within the Centre for Computational Engineering at Technische Universität Darmstadt.

References

1. Kirby, G. A., et al. "First cold powering test of REBCO roebel wound coil for the EuCARD2 future magnet development project." *IEEE Transactions on Applied Superconductivity* 27.4 (2017): 1-7.
2. Bortot, L., et al. "A Coupled A–H Formulation for Magneto-Thermal Transients in High-Temperature Superconducting Magnets." *IEEE Transactions on Applied Superconductivity* 30.5 (2020): 1-11.
3. Bortot, L., et al. "Numerical Analysis of the Screening Current-Induced Magnetic Field in the HTS Insert Dipole Magnet Feather-M2. 1-2." *Superconductor Science and Technology*, in press.